Journal of Engineering Physics and Thermophysics, Vol. 66, No. 4, 1994

STATISTICAL DESCRIPTION OF A TURBULENT GAS SUSPENSION FLOW OF LARGE PARTICLES COLLIDING WITH CHANNEL WALLS

I. V. Derevich

UDC 532.529

Based on the particle distribution density function method over space, velocity, and angular rotational velocity, a closed system of equations for the first and second moments of the fluctuations of the characteristics of the particles and boundary conditions representing the process of the loss of momentum and onset of rotation of the particles caused by collisions with walls are found.

In calculating pneumatic transport systems with turbulent gas suspension flows of large particles whose time of dynamic relaxation is much greater than the integrated time scale of turbulence, it is necessary to take into account the collisions of the particles with the walls of the channels. The intensity and frequency of collisions determine the rate of erosion of the internal surfaces of the tubes and the degree of the inverse effect of a disperse admixture on the carrier flow characteristics [1-4]. Investigation of the turbulent flow of inertial particles is also of interest in calculations of the characteristics of disperse flows emerging from nozzles in the form of jets [5].

Numerical simulation of gas suspension flows is based on the use of three approaches. First, it is based on a mixed description of a two-phase flow, Eulerian for the liquid phase and Lagrangian for particles. The averaged characteristics of the flow are found from calculations of several thousand Lagrangian trajectories realizable in a given random field of carrier phase velocity fluctuations (see, e.g., [6 –10]). In this case, determination of the inverse effect of the admixture on the carrier flow with account for the collision of the particles with the walls may require unrealistic expenditures of computer time. In the second approach, so far confirmed for relatively simple flows [11], both the fluctuational characteristics of the continuous and disperse phases and the distortion of the carrier flow turbulence under the action of the particles are determined on the basis of direct stochastic simulation. The likelihood that this procedure can be implemented at present to calculate pneumatic transport systems is highly questionable.

The third technique of numerical simulation of turbulent gas-disperse flows, based on a single Eulerian description of the dynamics of the gas phase and the particles, is the most economical one for the considered class of flows and provides averaged information that is valuable in practice. To perform specific calculations, it is necessary to have a closed system of equations for the moments of disperse phase velocity fluctuations and corresponding boundary conditions that represent the process of the collision of the particles with the bounding surface. Note that due to the collision of the particles with the walls the former acquire dynamic properties that affect the flow of the stream as a whole. In particular, the intensity of the pulsational motion of the inertial particles is determined not only by the degree of involvement of the admixture in the turbulent motion of the liquid phase, but also by the character of the interaction of the particles with the channel walls. When particles collide with the surface, a loss of momentum occurs, and the particles begin to rotate about their axes. The Magnus force appearing due to the rotation of the particles causes intense transverse motion of admixture particles [6, 7, 12, 13]. Thus, the channel walls along which a gas suspension is transported serve as a "positive feedback" in the gas-particles system leading to additional generation of disperse phase fluctuations compared to turbulent flow outside bounding surfaces.

In [14], on the basis of the Eulerian description, calculations were made for the effect of the swirling of particles on the characteristics of flow emerging from a nozzle. The profiles of the angular velocity of the rotation

---

G. M. Krzhizhanovskii Power Engineering Scientific-Research Institute, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 66, No. 4, pp. 387-397, April, 1994. Original article submitted April 10, 1992.

of the particles around their axes are specified empirically from the condition of the coincidence between the results of calculation of the motion of the particles and experimental data. The model did not involve boundary conditions for the equations of the characteristics of the particles. In [13] calculations of flow were performed on the basis of a system of equations for second and third moments of velocity fluctuations of particles closed on the basis of the hypothesis of quasi-normality. A system of boundary conditions was written in [13] in a form in which for calculations information is required not only about the coefficients of restitution of the momentum, but also about the value of the ratio between the angular velocities of the particles before and after collision with a wall as a function of the Stokes number (this ratio is also approximated from an analysis of experimental data). The necessity of resorting to additional semiempirical information, whose validity is reflected by the modeled class of flows, sharply narrows the predicting properties of the models [13, 14].

The aim of the present work is to develop an Eulerian technique for describing the dynamics of a turbulent gas suspension flow on the basis of a system of equations for moments and boundary conditions in which the process of the collision of particles is governed only by the coefficients of the recovery of momentum in collision. To solve the problem, an equation for the probability density function (PDF) of the distribution of particles in space, velocities, and angular rotational velocities is invoked.

In this work we do not discuss the problem of the effect of the gas suspension on the characteristics of gas flow. We consider a gas suspension flow in which the volumetric concentration of particles is low and the collisions of particles with one another can be neglected.

Using the method of functional differentiation, we construct a closed equation for the PDF; on the basis of this equation we obtain a system of equations for the first and second moments of fluctuations of the velocity and angular rotational velocity of the particles. We develop a method of approximate solution of the kinetic equation and obtain closed expressions that describe turbulent transfer of the mass, momentum, angular momentum, and pulsational energy of the disperse phase. From relations representing the process of interaction of a single particle with the bounding surface we find boundary conditions for a closed system for the first and second moments of the fluctuations of velocity and angular rotational velocity of an admixture particle.

1. With account for the Magnus force, the equations of motion of a single solid particle have the form [12]

$$\frac{dR_{pi}}{dt} = V_{pi}(t), \quad \frac{dV_{pi}}{dt} = \frac{1}{\tau}(U_i(\mathbf{R}_p, t) - V_{pi}) - \gamma_\omega \varepsilon_{kij} \Omega_{pk} (U_j(\mathbf{R}_p, t) - V_{pj}),$$

$$\frac{d\Omega_{pi}}{dt} = -\frac{1}{\tau_\omega} \Omega_{pi}(t). \tag{1}$$

We assume that the times of dynamic relaxation and the parameter γ_ω depend on the angular rotational velocity of the particles and the relative velocity of the averaged gas flow around a particle [15, 16].

In the equation for the angular velocity of the particles (1), the rotation of a particle due to flow vorticity is not taken into account. The possibility of this approximation for inertial particles is supported by the following arguments. Due to the inertia of the admixture, the characteristics of the particles do not depend on the local properties of the carrier medium, but are subjected to overall averaging over a region with a scale of the order of $l \sim \tau\sigma^{1/2}$ (σ is the level of the pulsational energy of the inertial particles with $\tau >> T_E$, $\sigma \sim ET_E/\tau$; E is the level of the pulsational energy of the carrier phase; T_E is the time macroscale of turbulent fluctuations of the gas). In this situation $l \sim E^{1/2}(T_E\tau)^{1/2}$ can exceed the characteristic transverse dimension of the channel. The angular rotational velocity of the particles due to the velocity gradient of the transverse flow is on the order of U/R. However, in reality the angular rotational velocity of the particles does not reach this value, since the time between two successive collisions of particles with the walls $t_c \sim R/\sigma^{1/2}$ is smaller than $\tau_\omega \sim \tau$ when $l > R$. The angular rotational velocity of the particles acquired due to collision with the surface is equal to about V_w/d_p (V_w is the characteristic velocity of the particles on the wall, $V_w \sim U$ [1–5]). Thus, the ratio between the angular rotational velocity due to flow vorticity and the angular rotational velocity due to the collision of particles with the wall is smaller than $d_p/R << 1$.

Let us introduce the PDF of the particles in space, velocities, and angular rotational velocities

$$\langle \Phi(\mathbf{x}, \mathbf{V}, \mathbf{\Omega}, t) \rangle = \langle \delta(\mathbf{x} - \mathbf{R}_p) \delta(\mathbf{V} - \mathbf{V}_p) \delta(\mathbf{\Omega} - \mathbf{\Omega}_p) \rangle . \quad (2)$$

Differentiation of Eq. (2) with respect to time and account for the equation of motion of a single particle (1) yield

$$\frac{\partial \langle \Phi \rangle}{\partial t} + V_k \frac{\partial \langle \Phi \rangle}{\partial x_k} + \frac{\partial}{\partial V_k} \left[\frac{(\langle U_k \rangle - V_k)}{\tau} \langle \Phi \rangle + \gamma_\omega \varepsilon_{ijk} \Omega_i (\langle U_j \rangle - V_j) \langle \Phi \rangle \right] -$$

$$- \frac{\partial}{\partial \Omega_k} \frac{\Omega_k}{\tau_\omega} \langle \Phi \rangle = - \frac{\partial}{\partial V_k} \frac{\langle u_k \Phi \rangle}{\tau} - \varepsilon_{ijk} \Omega_i \frac{\partial}{\partial V_k} \gamma_\omega \langle u_j \Phi \rangle . \quad (3)$$

To close Eq. (3), it is necessary to find an expression for the correlator $\langle u_j \Phi \rangle$. For this purpose, we use the Furutsu-Novikov formula [17]:

$$\langle u_j \Phi \rangle = \int d\mathbf{x}_1 \int_0^t d\xi \langle u_j(\mathbf{x}, t) u_l(\mathbf{x}_1, \xi) \rangle \langle \frac{\delta \Phi(\mathbf{x}, \mathbf{V}, \mathbf{\Omega}, t)}{\delta u_l(x_1, \xi)} \rangle ,$$

$$\frac{\delta \Phi(\mathbf{x}, \mathbf{V}, \mathbf{\Omega}, t)}{\delta u_l(\mathbf{x}_1, \xi)} = - \frac{\partial \Phi}{\partial x_m} \frac{\delta R_{pm}(t)}{\delta u_l(\mathbf{x}_1, \xi)} - \frac{\partial \Phi}{\partial V_m} \frac{\delta V_{pm}(t)}{\delta u_l(\mathbf{x}_1, \xi)} - \frac{\partial \Phi}{\partial \Omega_m} \frac{\delta \Omega_{pm}(t)}{\delta u_l(\mathbf{x}_1, \xi)} , \quad (4)$$

where $\delta\Phi / \delta u_l$ is a functional derivative.

For an inertial admixture the characteristic time scale for the correlations of the fluctuations of velocity and angular rotational velocity of the particles is on the order of $\tau \sim \tau_\omega$. The characteristic radius for the correlations of the gas velocity fluctuations is $\sim T_E$. A substantial change in the functional derivatives in Eqs. (4) is observed on the scales $(t-\xi) \sim \tau \sim \tau_\omega$ [18]. In this case Eqs. (4) involve a small parameter T_E / τ, which can be used for approximate calculation of functional derivatives. In the first approximation in this parameter of smallness, we may assume that $T_E = 0$ in Eq. (6), which is equivalent to the replacement of the real correlational tensor of the gas velocity fluctuations by the effective one [17]

$$\langle u_i(\mathbf{x}_1, t) u_j(\mathbf{x}_2, t+s) \rangle = T_E \langle u_i(\mathbf{x}_1, t) u_j(\mathbf{x}_2, t) \rangle \delta(s) .$$

The introduction of the effective tensor of the correlations of liquid phase velocity fluctuations corresponds to passage to a Gaussian delta-time-correlated random process that approximates random effects of the fluctuation of the medium velocities on the inertial system of particles.

The functional derivative of the angular rotational velocity of the particles is equal to zero because the equation for the angular rotational velocity (1) does not explicitly include the function $u_i(x,t)$. The functional derivative of the radius vector of a particle also does not enter into Eq. (4) when $\xi \to t$ [18].

To calculate the functional derivative of the velocity of a particle, we write down Eq. (1) in an integral form

$$V_{pm}(t) = \int_0^t \frac{ds}{\tau} \exp\left(-\frac{t-s}{\tau}\right) [U_m(\mathbf{R}_p(s), s) + \tau\gamma_\omega \varepsilon_{kjm} (U_j(\mathbf{R}_p(s), s) - V_{pj}(s)) \Omega_{pk}(s)] . \quad (5)$$

Applying to Eq. (5) the operator of functional differentiation $\delta/\delta u_l(\mathbf{x}_1, \xi)$ and taking into account the equality $\delta u_j(\mathbf{x}, t)/\delta u_l(\mathbf{x}, \xi) = \delta_{jl}\delta(\mathbf{x}-\mathbf{x}_1)\delta(t-\xi)$, we obtain an expression for the correlator $\langle u_j \Phi \rangle$:

$$\langle u_j \Phi \rangle = - \frac{T_E}{\tau} \langle u_j u_l \rangle \frac{\partial \langle \Phi \rangle}{\partial V_l} - T_E \gamma_\omega \langle u_j u_l \rangle \varepsilon_{kln} \Omega_k \frac{\partial \Phi}{\partial V_n} . \quad (6)$$

Substituting Eq. (6) into Eq. (3), we write a closed equation for PDF of the distribution of the particles in space, velocities, and angular rotational velocities

$$\frac{\partial \langle \Phi \rangle}{\partial t} + V_k \frac{\partial \langle \Phi \rangle}{\partial x_k} + \frac{\partial}{\partial V_k} \times \left[\frac{(\langle U_k \rangle - V_k)}{\tau} \langle \Phi \rangle + \gamma_\omega \varepsilon_{ijk} \Omega_i (\langle U_j \rangle - V_j) \langle \Phi \rangle \right] -$$

$$-\frac{\partial}{\partial \Omega_k}\frac{\Omega_k}{\tau_\omega}\langle \Phi \rangle = \frac{\partial^2}{\partial V_j \partial V_k} \times$$

$$\times \left[\left(\frac{T_E}{\tau^2}\langle u_j u_k \rangle + \frac{2T_E}{\tau}\gamma_\omega \varepsilon_{ink} \Omega_i \langle u_j u_n \rangle + \gamma_\omega^2 T_E \varepsilon_{nlj} \varepsilon_{imk} \langle u_m u_l \rangle \Omega_n \Omega_i \right) \langle \Phi \rangle \right]. \tag{7}$$

Equation (7) is similar to the Fokker-Planck equation. It describes the onset of turbulent motion in a disperse phase as a result of pulsations of the force of viscous resistance and the Magnus force. We note that the intensity of pulsational motion, as is seen from Eq. (7), depends explicitly on the intensity of the rotation of the particles.

2. Applying ensemble averaging of turbulent realizations, we find the averaged number density, velocity, and angular rotational velocity of the particles

$$\langle N(\mathbf{x}, t) \rangle = \int d\mathbf{V} \int d\Omega \langle \Phi(\mathbf{x}, \mathbf{V}, \Omega, t) \rangle,$$

$$\langle V_i(\mathbf{x}, t) \rangle \langle N \rangle = \int d\mathbf{V} \int d\Omega\, V_i \langle \Phi(\mathbf{x}, \mathbf{V}, \Omega, t) \rangle,$$

$$\langle \Omega_i(\mathbf{x}, t) \rangle \langle N \rangle = \int d\mathbf{V} \int d\Omega\, \Omega_i \langle \Phi(\mathbf{x}, \mathbf{V}, \Omega, t) \rangle.$$

In Eq. (7) the times of dynamic relaxation of the particles τ, τ_ω and the parameter γ_ω depend on the velocity of the averaged flow of gas around the particles and the angular rotational velocity. In what follows we take into account the effect of just the averaged relative slip of the phases $<U>-<V>$ and the averaged angular velocity of the rotation of the particles about their axes $<\Omega>$ on these parameters.

For subsequent analysis of kinetic equation (7) we go over to the variables $v_i = V_i - <V_i>$, $\omega_i = \Omega_i - <\Omega_i>$ (v_i, ω_i are the fluctuations of the velocity and angular rotational velocity of the disperse phase):

$$\frac{D\langle \Phi \rangle}{Dt} + \left\{ -\frac{D\langle V_k \rangle}{Dt} + \frac{\langle U_k \rangle - \langle V_k \rangle}{\tau} + \gamma_\omega \varepsilon_{ijk} \times \right.$$

$$\times [\langle \Omega_i \rangle (\langle U_j \rangle - \langle V_j \rangle) - \langle \Omega_i \rangle v_j + \omega_i (\langle U_j \rangle - \langle V_j \rangle) -$$

$$\left. - \omega_i v_j] \right\} \frac{\partial \langle \Phi \rangle}{\partial v_k} - \left(\frac{D\langle \Omega_k \rangle}{Dt} + \frac{\langle \Omega_k \rangle}{\tau_\omega} \right) \frac{\partial \langle \Phi \rangle}{\partial \omega_k} -$$

$$- v_k \frac{\partial \langle V_i \rangle}{\partial x_k} \frac{\partial \langle \Phi \rangle}{\partial v_i} - v_k \frac{\partial \langle \Omega_i \rangle}{\partial x_k} \frac{\partial \langle \Phi \rangle}{\partial \omega_i} - \frac{1}{\tau} \frac{\partial}{\partial v_k} v_k \langle \Phi \rangle -$$

$$- \frac{1}{\tau_\omega} \frac{\partial}{\partial \omega_k} \omega_k \langle \Phi \rangle = \left[\frac{T_E}{\tau^2} \langle u_j u_k \rangle + \frac{2T_E}{\tau} \gamma_\omega \varepsilon_{ink} \times \right.$$

$$\times (\langle \Omega_i \rangle + \omega_i) \langle u_j u_n \rangle + T_E \gamma_\omega^2 \varepsilon_{nlj} \varepsilon_{imk} \langle u_m u_l \rangle \times$$

$$\left. \times (\langle \Omega_n \rangle + \omega_n)(\langle \Omega_i \rangle + \omega_i) \right] \frac{\partial^2 \langle \Phi \rangle}{\partial v_j \partial v_k}, \tag{8}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \langle V_k \rangle \frac{\partial}{\partial x_k}.$$

Kinetic equation (8) yields a system of equations for the averaged concentrations, velocity, and angular rotational velocity of the particles

$$\frac{\partial \langle N \rangle}{\partial t} + \frac{\partial}{\partial x_k} \langle N \rangle \langle V_k \rangle = 0 , \tag{9}$$

$$\frac{D \langle V_i \rangle}{Dt} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle v_i v_k \rangle}{\partial x_k} = \frac{\langle U_i \rangle - \langle V_i \rangle}{\tau} + \gamma_\omega \varepsilon_{ijk} \times$$

$$\times [\langle \Omega_k \rangle (\langle U_j \rangle - \langle V_j \rangle) - \langle \omega_k v_j \rangle] , \tag{10}$$

$$\frac{D \langle \Omega_i \rangle}{Dt} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle \omega_i v_k \rangle}{\partial x_k} = - \frac{\langle \Omega_i \rangle}{\tau_\omega} , \tag{11}$$

where $<v_i v_k><N> = \int d\mathbf{v} \int d\omega\, v_i v_k <\Phi>$, $<\omega_i v_k><N> = \int d\mathbf{v} \int d\omega\, \omega_i v_k <\Phi>$ are the turbulent fluxes of momentum and angular momentum arising due to the pulsational motion of the particles. On the basis of Eq. (8) we write a system of equations for the second moments of the fluctuations of the velocity and angular rotational velocity of the particles

$$\frac{D \langle v_i v_j \rangle}{Dt} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle v_i v_j v_k \rangle}{\partial x_k} + \langle v_i v_k \rangle \frac{\partial \langle V_j \rangle}{\partial x_k} +$$

$$+ \langle v_j v_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k} = \frac{2}{\tau} (\sigma^0_{ij} - \langle v_i v_j \rangle) , \tag{12}$$

$$\sigma^0_{ij} = \frac{T_E}{\tau} \langle u_i u_j \rangle - \frac{\tau \gamma_\omega}{2} \left\{ \varepsilon_{lmj} \left[(\langle U_m \rangle - \langle V_m \rangle) \langle \omega_l v_i \rangle + \right.\right.$$

$$\left. + \langle \Omega_l \rangle \left(\frac{T_E}{\tau} \langle u_m u_i \rangle - \langle v_m v_i \rangle \right) - \langle \omega_l v_m v_i \rangle \right] +$$

$$+ \varepsilon_{lmi} \left[(\langle U_m \rangle - \langle V_m \rangle) \langle \omega_l v_j \rangle + \langle \Omega_l \rangle \left(\frac{T_E}{\tau} \langle u_m u_j \rangle - \right.\right.$$

$$\left.\left.\left. - \langle v_m v_j \rangle \right) - \langle \omega_l v_m v_j \rangle \right]\right\} + \frac{T_E \tau \gamma_\omega^2}{2} \varepsilon_{nlj} \varepsilon_{kmi} \times$$

$$\times \langle u_m u_l \rangle (\langle \Omega_n \rangle \langle \Omega_k \rangle + \langle \omega_n \omega_k \rangle) ,$$

$$\frac{D \langle \omega_i \omega_j \rangle}{Dt} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle \omega_i \omega_j v_k \rangle}{\partial x_k} + \langle \omega_i v_k \rangle \frac{\partial \langle \Omega_j \rangle}{\partial x_k} +$$

$$+ \langle \omega_j v_k \rangle \frac{\partial \langle \Omega_i \rangle}{\partial x_k} = - \frac{2}{\tau_\omega} \langle \omega_i \omega_j \rangle , \tag{13}$$

$$\frac{D \langle \omega_i v_j \rangle}{Dt} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle \omega_i \omega_j v_k \rangle}{\partial x_k} + \langle \omega_i v_k \rangle \frac{\partial \langle V_j \rangle}{\partial x_k} +$$

$$+ \langle v_j v_k \rangle \frac{\partial \langle \Omega_i \rangle}{\partial x_k} - \gamma_\omega \varepsilon_{lmj} [\langle \omega_l \omega_i \rangle (\langle U_m \rangle - \langle V_m \rangle) -$$

$$- \langle \Omega_l \rangle \langle v_m \omega_i \rangle - \langle \omega_l \omega_i v_m \rangle] = - \left(\frac{1}{\tau} + \frac{1}{\tau_\omega} \right) \langle \omega_i v_j \rangle , \quad (14)$$

$$\langle v_i v_j v_k \rangle \langle N \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, v_i v_j v_k \langle \Phi \rangle ,$$

$$\langle \omega_i \omega_j v_k \rangle \langle N \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, \omega_i \omega_j v_k \langle \Phi \rangle ,$$

$$\langle \omega_i v_j v_k \rangle \langle N \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, \omega_i v_j v_k \langle \Phi \rangle ,$$

where $<v_i v_j v_k>$, $<\omega_i \omega_j \omega_k>$, $<\omega_i v_j v_k>$ are the third moments of the fluctuations of the velocity and angular rotational velocity of the particles.

3. The system of equations (9)–(14) is not closed; the equations for the first and second moments of the fluctuations include expressions for higher-order moments. The closure of the system of equations for the second moments on the basis of equations for the third moments of the fluctuations of the disperse phase velocity require the use of the hypothesis of quasi-normality when expanding fourth-order correlations [19]. In the present work, to close the system of equations for the first and second moments of the fluctuations of the disperse phase characteristics and construct boundary conditions we suggest a more general approach based on an approximate solution of kinetic equation (8). Equation (8) is stated in the form

$$\frac{D\langle \Phi \rangle}{Dt} + \left\{ - \frac{D\langle V_k \rangle}{Dt} + \frac{\langle U_k \rangle - \langle V_k \rangle}{\tau} + \gamma_\omega \varepsilon_{ijk} \times \right.$$

$$\times [\langle \Omega_i \rangle (\langle U_j \rangle - \langle V_j \rangle) - \langle \Omega_i \rangle v_j + \omega_i (\langle U_j \rangle - \langle V_j \rangle) -$$

$$\left. - \omega_i v_j] \right\} \frac{\partial \langle \Phi \rangle}{\partial v_k} - \left(\frac{D\langle \Omega_k \rangle}{Dt} + \frac{\langle \Omega_k \rangle}{\tau_\omega} \right) \frac{\partial \langle \Phi \rangle}{\partial \omega_k} -$$

$$- v_k \frac{\partial \langle V_i \rangle}{\partial x_k} \frac{\partial \langle \Phi \rangle}{\partial v_i} - v_k \frac{\partial \langle \Omega_i \rangle}{\partial x_k} \frac{\partial \langle \Phi \rangle}{\partial \omega_i} - \frac{1}{\tau} (1 - \delta_{ik}) \overset{0}{\sigma}_{ik} \frac{\partial^2 \langle \Phi \rangle}{\partial v_i \partial v_k} + \frac{\sigma_{ii} - \overset{0}{\sigma}_{ii}}{\tau} \frac{\partial^2 \langle \Phi \rangle}{\partial v_i \partial v_i} = A \langle \Phi \rangle , \quad (15)$$

$$A = \frac{1}{\tau} \left(\sigma_{ii} \frac{\partial^2}{\partial v_i \partial v_i} + \frac{\partial}{\partial v_i} v_i \right) + \frac{1}{\tau_\omega} \frac{\partial}{\partial \omega_i} \omega_i , \quad \sigma_{ii} = \langle v_i v_i \rangle ,$$

where the operator A on the right-hand side describes the process of the onset and decay of the pulsational motion of the disperse admixture as a result of the interaction of the particles with turbulent moles. The method for solving kinetic Eq. (15) is analogous to the Chapman-Enskog method [20] applied in the kinetic theory of gases. In the flow of the inertial particles a characteristic spatial scale – the length of the inertial path $l \sim \tau \sigma^{1/2}$ – arises. Here, the characteristic scale of the change in the averaged quantities is $L > (l+R)$. In this case, the small parameter $\varepsilon \sim l/L < 1$ appears on the left-hand side of Eq. (15). This allows one to seek the solution of Eq. (15) in the form

$$\langle \Phi \rangle = \langle \Phi_0 \rangle + \langle \Phi_1 \rangle , \quad A \langle \Phi_0 \rangle = 0 ,$$

where the correction $<\Phi_1>$ is linear in the gradients of the averaged parameters of the disperse phase and satisfies the normalization conditions

$$\int d\mathbf{v} \int d\boldsymbol{\omega} \langle \Phi_1 \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, v_i \langle \Phi_1 \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, \omega_i \langle \Phi_1 \rangle =$$

$$= \int d\mathbf{v} \int d\boldsymbol{\omega} \, \omega_i \omega_j \langle \Phi_1 \rangle = \int d\mathbf{v} \int d\boldsymbol{\omega} \, v_i v_i \langle \Phi_1 \rangle = 0 .$$

In the flow core there is no source that would directly generate fluctuations of the rotational velocity of the particles (the rotation of the particles sets in only as a result of collision with the channel walls). The zero approximation in Eq. (15) has the form

$$\langle \Phi_0 (\mathbf{x}, \mathbf{v}, \boldsymbol{\omega}, t) \rangle = \langle N(\mathbf{x}, t) \rangle \prod_{i=1}^{3} (2\pi \sigma_{ii})^{-1/2} \times \exp\left(-\frac{v_i^2}{2\sigma_{ii}} \right) \delta(\boldsymbol{\omega}) = \varphi_0 \, \delta(\boldsymbol{\omega}) \langle N \rangle . \tag{16}$$

Using the zero approximation (16) to close system of equations (9)–(14) and assuming that the PDF depends on the coordinates and the time only via the averaged parameters of the flow of the particles, we find an approximate solution of Eq. (15). This solution is linear in the gradients of the averaged parameters of the disperse phase:

$$\langle \Phi (\mathbf{x}, \mathbf{v}, \boldsymbol{\omega}, t) \rangle = \langle N(\mathbf{x}, t) \rangle \varphi_0 \left\{ 1 + \frac{1}{2} \overset{0}{\sigma}_{ik} (1 - \delta_{ik}) \times \right.$$

$$\times \frac{v_i v_k}{\sigma_{ii} \sigma_{kk}} - \frac{\tau}{2\sigma_{ii}} (v_i v_k - \delta_{ik} v^2) \frac{\partial \langle V_i \rangle}{\partial x_k} -$$

$$\left. - \frac{\tau}{3} v_k \left[\frac{v_i^2}{2\sigma_{ii}} - \left(\frac{1}{2} + \delta_{ik} \right) \right] \frac{\partial \ln \sigma_{ii}}{\partial x_k} \right\} \delta(\omega) +$$

$$+ \langle N(\mathbf{x}, t) \rangle \varphi_0 \left(\frac{1}{\tau} + \frac{1}{\tau_\omega} \right)^{-1} v_k \frac{\partial \langle \Omega_i \rangle}{\partial x_k} \frac{\partial \delta(\boldsymbol{\omega})}{\partial \omega_i} . \tag{17}$$

Formula (17) permits one to calculate expressions that represent pulsational transfer of the momentum, angular momentum, and intensity of turbulent pulsations of the dispersed phase:

$$\langle v_i v_j \rangle = \sigma_{ii} \delta_{ij} + (1 - \delta_{ij}) \overset{0}{\sigma}_{ij} - \frac{\tau}{2} \left[\sigma_{ii} \frac{\partial \langle V_j \rangle}{\partial x_i} + \right.$$

$$\left. + \sigma_{jj} \frac{\partial \langle V_i \rangle}{\partial x_j} - \frac{2}{3} \delta_{ij} \sigma_{kk} \frac{\partial \langle V_k \rangle}{\partial x_k} \right] ,$$

$$\langle v_i \omega_j \rangle = - \left(\frac{1}{\tau} + \frac{1}{\tau_\omega} \right)^{-1} \langle v_i v_k \rangle \frac{\partial \langle \Omega_j \rangle}{\partial x_k} , \quad \langle \omega_i \omega_j \rangle = 0 , \tag{18}$$

$$\langle v_i \omega_j \omega_k \rangle = 0 , \quad \langle v_i v_j v_k \rangle = - \delta_{ij} \frac{1}{3} (2\delta_{ik} + \delta_{ii}) \tau \sigma_{kk} \frac{\partial \sigma_{ii}}{\partial x_k} ,$$

$$\langle \omega_l v_m v_j \rangle = - \left(\frac{1}{\tau} + \frac{1}{\tau_\omega} \right)^{-1} \langle v_k v_m v_j \rangle \frac{\partial \langle \Omega_l \rangle}{\partial x_k} .$$

Expressions (18) and system of equations (9)–(14) represent a closed system of equations for calculating the first and seconds moments of the fluctuations of the velocity and angular rotational velocity of the particles.

4. The collision of particles with a surface is a complex process accompanied by deformation of the particle and the wall, evolution of heat, and onset of rotation of the particle about the point of contact with the surface. We consider a model of a collision of a disperse admixture with a channel wall in which the reflected particle losses momentum along the y and z axes (the y axis is directed along the normal to the surface, the x axis coincides with

the flow direction) and the remaining velocities and angular rotational velocities of a particle before and after the collision are interrelated as [12, 13]

$$V_x'' = \alpha_1 V_x' + \alpha_2 \Omega_z', \quad \Omega_z'' = \beta_1 V_x' + \beta_2 \Omega_z', \quad V_y'' = -k_n V_y', \quad V_z'' = k_t V_z',$$
$$\alpha_1 = \frac{5 + 2k_t}{7}, \quad \alpha_2 = -d_p \frac{1 - k_t}{7}, \quad \beta_1 = -\frac{10(1 - k_t)}{7 d_p}, \quad \beta_2 = \frac{5k_t + 2}{7}, \tag{19}$$

where k_n and k_t are the coefficients of restitution of the momentum of the particles after impact against the surface.

The PDF of particles near the surface is written in the boundary layer approximation $\partial\langle V_x\rangle/\partial y >> \partial\langle V_x\rangle/\partial x$:

$$\langle \Phi(\mathbf{x}, \mathbf{v}, \boldsymbol{\omega}, t)\rangle = \langle N\rangle \varphi_0 \left\{ 1 + \Sigma_{xy} \frac{v_x v_y}{\sigma_{xx}\sigma_{yy}} - \right.$$
$$\left. - \frac{\tau}{3} v_y \left[\frac{v_i^2}{2\sigma_{ii}} - \left(\frac{1}{2} + \delta_{iy} \right) \right] \frac{\partial \ln \sigma_{ii}}{\partial y} \right\} \delta(\omega_z) + \langle N\rangle \varphi_0 \left(\frac{1}{\tau} + \frac{1}{\tau_\omega} \right)^{-1} v_y \frac{\partial \langle \Omega_z\rangle}{\partial y} \frac{\partial \delta(\omega_z)}{\partial \omega_z}, \tag{20}$$

$$\Sigma_{xy} = \sigma_{xy}^0 - \frac{\tau}{2} \sigma_{yy} \frac{\partial \langle V_x\rangle}{\partial y}.$$

The collision of particles with a wall causes a transformation of the PDF of the particles incident on the wall ($V_y < 0$). The PDF of the reflected particles is calculated from the PDF of the incident particles according to the relation

$$\langle \Phi_+(\mathbf{x}, \mathbf{V}'', \boldsymbol{\Omega}'', t)\rangle - \int_{-\infty}^{\infty} dV_x' \int_{-\infty}^{\infty} dV_y' \int_{-\infty}^{\infty} dV_z' \int_{-\infty}^{\infty} d\Omega_z' \times$$
$$\times G(\mathbf{V}'', \boldsymbol{\Omega}''; \mathbf{V}', \boldsymbol{\Omega}') \langle \Phi(\mathbf{x}, \mathbf{V}', \boldsymbol{\Omega}', t)\rangle, \tag{21}$$

where the transformation kernel G is determined from the boundary relations for a single particle (19):

$$G(\mathbf{V}'', \boldsymbol{\Omega}''; \mathbf{V}', \boldsymbol{\Omega}') = \delta(V_z'' - k_t V_z') \delta(V_y'' + k_n V_y') \times$$
$$\times \delta(\beta_2 V_x'' - \alpha_2 \Omega_z'' - k_t V_x') \delta(\alpha_1 \Omega_z'' - \beta_1 V_x'' - k_t \Omega_z'). \tag{22}$$

Expressions (21) and (22) yield the equality of the number of particles incident on and reflected from the channel wall

$$\langle N_+\rangle = \int_{-\infty}^{\infty} dV_x'' \int_0^{\infty} dV_y'' \int_{-\infty}^{\infty} dV_z'' \int_{-\infty}^{\infty} d\Omega_z'' \langle \Phi_+(\mathbf{x}, \mathbf{V}'', \Omega'', t)\rangle =$$
$$= \int_{-\infty}^{\infty} dV_x' \int_{-\infty}^{0} dV_y' \int_{-\infty}^{\infty} dV_z' \int_{-\infty}^{\infty} d\Omega_z' \langle \Phi(\mathbf{x}, \mathbf{V}', \Omega', t)\rangle = \langle N_-\rangle.$$

Upon substituting Eq. (22) into (21), we find the PDF of the particles reflected from the channel wall

$$\langle \Phi_+(\mathbf{x}, \mathbf{v}'', \boldsymbol{\omega}'', t)\rangle = \langle N\rangle \varphi_0'' \left\{ 1 - \Sigma_{xy} \frac{v_x'' v_y''}{k_t k_n \sigma_{xx}\sigma_{yy}} + \right.$$

$$+\frac{\tau}{3}\frac{v_y''}{k_n}\left[\left(\frac{v_x''^2}{2k_t^2\sigma_{xx}}-\frac{1}{2}\right)\frac{\partial\ln\sigma_{xx}}{\partial y}+\left(\frac{v_y''^2}{2k_n^2\sigma_{yy}}-\frac{3}{2}\right)\times\right.$$

$$\left.\left.\times\frac{\partial\ln\sigma_{yy}}{\partial y}+\left(\frac{v_z''^2}{2k_t^2\sigma_{zz}}-\frac{1}{2}\right)\frac{\partial\ln\sigma_{zz}}{\partial y}\right]\right\}\frac{1}{k_t}\delta\left(\frac{\omega_z''}{k_t}\right)-$$

$$-\langle N\rangle\varphi_0''\left(\frac{1}{\tau}+\frac{1}{\tau_\omega}\right)^{-1}\frac{v_y''}{k_n}\frac{\partial\langle\Omega_z\rangle}{\partial y}\frac{\partial}{\partial\omega_z''}\delta\left(\frac{\omega_z''}{k_t}\right), \qquad (23)$$

$$\varphi_0''=(2\pi\sigma_{xx}k_t^2)^{-1/2}\exp\left(-\frac{v_x''^2}{2k_t^2\sigma_{xx}}\right)(2\pi\sigma_{yy}k_n^2)^{-1/2}\times$$

$$\times\exp\left(-\frac{v_y''^2}{2k_n^2\sigma_{yy}}\right)(2\pi\sigma_{zz}k_t^2)^{-1/2}\exp\left(-\frac{v_z''^2}{2k_t^2\sigma_{zz}}\right),$$

$$v_x''=\beta_2V_x''-\alpha_2\Omega_z''-k_t\langle V_x\rangle,\quad v_y''=V_y''+k_n\langle V_y\rangle,$$

$$\omega_z''=\alpha_1\Omega_z''-\beta_1V_x''-k_t\langle\Omega_z\rangle,\quad v_z''=V_z''.$$

Boundary conditions for the averaged concentration, velocity, angular rotational velocity, and intensity of turbulent fluctuations of the particles can be found from the condition of equality between the sum of the fluxes of the corresponding dynamic characteristics Q of particles incident and reflected from the surface and the stream in the flow

$$J\{Q\}=J_-\{Q\}+J_+\{Q\},\quad J\{Q\}=\int d\mathbf{V}\int_{-\infty}^{\infty}d\Omega_z\,V_y\,Q\langle\Phi(\mathbf{x},\mathbf{V},\boldsymbol{\Omega},t)\rangle,$$

$$J_-\{Q\}=\int_{-\infty}^{\infty}dV_x\int_{-\infty}^{0}dV_y\int_{-\infty}^{\infty}dV_z\int_{-\infty}^{\infty}d\Omega_z\,V_y\,Q\langle\Phi(\mathbf{x},\mathbf{V},\boldsymbol{\Omega},t)\rangle,$$

$$J_+\{Q\}=\int_{-\infty}^{\infty}dV_x''\int_{0}^{\infty}dV_y''\int_{-\infty}^{\infty}dV_z''\int_{-\infty}^{\infty}d\Omega_z''\,V_y''\,Q\langle\Phi_+(\mathbf{x},\mathbf{V}'',\boldsymbol{\Omega}'',t)\rangle.$$

Here J_-, J_+, J are the fluxes of the characteristics Q calculated for incident and reflected particles and the stream in the flow related to the channel surface.

We find a boundary condition for the normal component of the averaged velocity of the particles ($y=0$)

$$\langle V_y\rangle\left[1+\frac{1-k_n}{1+k_n}\operatorname{erf}\left(\frac{\langle V_y\rangle}{\sqrt{2\sigma_{yy}}}\right)\right]+$$

$$+\frac{1-k_n}{1+k_n}\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\exp\left(-\frac{\langle V_y\rangle^2}{2\sigma_{yy}}\right)=0. \qquad (24)$$

Relation (24) connects the intensity of the fluctuations in the velocity of the particles normal to the surface with the transverse velocity of the disperse phase. Using Eq. (10) to find the transverse velocity of the disperse phase, we come to a boundary condition for the concentration of particles which couples the gradient and the value for the concentration of the particles on the surface washed by the flow. Equation (24) yields the inequality $\langle V_y \rangle < \sigma_{yy}^{1/2}$. In this case, boundary condition (24) takes the form [21]

$$\langle V_y \rangle + \frac{1-k_n}{1+k_n}\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2} = 0 .$$

Similarly, we find boundary conditions for the averaged axial velocity and angular rotational velocity of the particles acquired due to collision with the surface

$$\frac{\tau\,\sigma_{yy}}{2}\frac{\partial\langle V_x\rangle}{\partial y} - \left[\langle V_y\rangle + \left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{A_1}{B}\right]\langle V_x\rangle = -\frac{2k_n\alpha_2}{B}\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\langle\Omega_z\rangle, \tag{25}$$

$$\left(\frac{1}{\tau}+\frac{1}{\tau_\omega}\right)^{-1}\sigma_{yy}\frac{\partial\langle\Omega_z\rangle}{\partial y} - \left[\langle V_y\rangle + \left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{A_2}{B}\right]\langle\Omega_z\rangle = -\frac{2k_n\beta_1}{B}\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\langle V_x\rangle, \tag{26}$$

$$A_1 = 1 - k_n^2k_t - 3/7k_n(1-k_t), \quad A_2 = 1 - k_n^2k_t + 3/7k_n(1-k_t),$$

$$B = 1 + k_n(1+k_t) + k_n^2k_t .$$

A boundary condition for the intensity of the fluctuations in the velocity of the particles normal to the surface washed by the flow is found in an analogous fashion from the condition of the balance of the fluxes $J\{v_y^2\}$. It has the form

$$\left[\langle V_y\rangle + 2\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{1-k_n^3}{1+k_n^3}\right]\sigma_{yy} = \tau\,\sigma_{yy}\frac{\partial\sigma_{yy}}{\partial y}. \tag{27}$$

The fluctuational component of the velocity of reflected particles that is directed along the averaged flow velocity can be determined as the difference between the actual velocity in the longitudinal direction after collision of the particles with the wall and the averaged velocity of the reflected particles

$$v_x = V_x'' - (\alpha_1\langle V_x\rangle + \alpha_2\langle\Omega_z\rangle) = \frac{\alpha_1 v_x'' + \alpha_2\omega_z''}{k_t}.$$

The boundary condition for the intensity of longitudinal fluctuations of the velocity of the particles takes the form

$$\left[\langle V_y\rangle + \left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{1-\alpha_1^2k_n}{1+\alpha_1^2k_n}\right]\sigma_{xx} = \frac{\tau}{3}\sigma_{yy}\frac{\partial\sigma_{xx}}{\partial y}. \tag{28}$$

Let us analyze boundary conditions (25)-(28). In the case of a perfectly smooth and elastic surface ($k_t = 1$, $k_n = 1$) we obtain $\partial\langle V_x\rangle/\partial y = \partial\langle\Omega_z\rangle/\partial y = \partial\sigma_{yy}/\partial y = \partial\sigma_{xx}/\partial y = 0$, i.e., the fluxes of momentum and angular momentum of proper rotation of the particles to the channel wall are absent. Moreover, as follows from the equation for the averaged rotational velocity of the particles (11), $\langle\Omega_z\rangle = 0$ in the flow. Rotation of particles sets in only in a rough channel ($k_t < 1$), either elastic ($k_n = 1$) or inelastic ($k_n < 1$). In the case of a perfectly elastic but rough surface the boundary conditions for the averaged velocity and angular rotational velocity take a simple form:

$$\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{1}{7}\frac{1-k_t}{1+k_t}(2\langle V_x\rangle + d_p\langle\Omega_z\rangle) = -\frac{\tau}{2}\sigma_{yy}\frac{\partial\langle V_x\rangle}{\partial y},$$

$$\left(\frac{2}{\pi}\sigma_{yy}\right)^{1/2}\frac{5}{7}\frac{1-k_t}{1+k_t}(2\langle V_x\rangle + d_p\langle\Omega_z\rangle) = -\left(\frac{1}{\tau}+\frac{1}{\tau_\omega}\right)^{-1}\sigma_{yy}d_p\frac{\partial\langle\Omega_z\rangle}{\partial y}.$$

The employment of the probability density function of the distribution of particles over coordinates, velocitites, and angular rotational velocities made it possible to construct a closed description of a class of flows having practical significance.

NOTATION

R_{pi}, V_{pi}, Ω_{pi}, radius vector, velocity, and angular rotational velocity of a particle; $U_i(\mathrm{x}, t)$, carrier flow velocity; τ, τ_ω, times of dynamic relaxation of the velocity and angular rotational velocity of particles; γ_ω, interaction parameter, proportional to the ratio of the densities of gas and particles; ε_{ijk}, antisymmetric tensor; R, characteristic transverse dimension of the channel; d_p, diameter of particles; T_E, characteristic time macroscale of turbulent fluctuations of the gas velocity; $\delta(\mathrm{x})$, Dirac delta function; $u_i(\mathrm{x},t)$, $v_i(\mathrm{x},t)$, fluctuations of the gas and disperse phase velocities; $\omega_i(\mathrm{x}, t)$, fluctuation of the angular rotational velocity of particles; $\mathrm{erf}(x) = 2/\sqrt{\pi}\int_0^x dt_{\exp}(-t^2)$, standard error function.

Angular brackets denote values obtained as a result of averaging over the ensemble of turbulent realizations; a single prime denotes values before collision with the wall; a double prime denotes values after collision with the wall.

REFERENCES

1. A. S. Mul'gi, Turbulent Two-Phase Flows [in Russian], Pt. 2, Tallinn (1979), pp. 47-53.
2. S. L. Lee and F. Durst, Int. J. Multiphase Flow, **8**, 125-128 (1988).
3. Y. Tsuji and Y. Morikawa, J. Fluid Mech., **120**, 385-409 (1982).
4. Y. Tsuji, Y. Morikawa, and H. Shiomi, J. Fluid Mech., **139**, 417-434 (1984).
5. Y. Tsuji, Y. Morikawa, T. Tanaka, et al., Int. J. Multiphase Flow, **14**, 565-574 (1988).
6. S. Matsumoto and S. Saito, J. Chem. Eng. Japan, **3**, 223-230 (1970).
7. S. Matsumoto, S. Saito, and S. Maeda, J. Chem. Eng. Japan, **10**, 23-28 (1976).
8. G. A. Kallio and M. W. Reeks, Int. J. Multiphase Flow, **15**, 443-446 (1989).
9. A. H. Govan, G. F. Hewitt, and C. F. Ngan, Int. J. Multiphase Flow, **15**, 471-481 (1989).
10. K. D. Squires and J. K. Eaton, J. Fluid Mech., **226**, 1-35 (1991).
11. K. D. Squires and J. K. Eaton, Phys. Fluids A, **2**, 1191-1203 (1990).
12. A. A. Shraiber, V. N. Milyutin, and V. P. Yatsenko, Hydromechanics of Two-Component Flows with Solid Polydisperse Material [in Russian], Kiev (1976).
13. L. V. Kondratiev and V. V. Shor, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 56-64 (1990).
14. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Flows of a Gas Suspension [in Russian], Kiev (1987).
15. S. C. Dennis, S. N. Singh, and D. B. Ingham, J. Fluid Mech., **101**, 257-279 (1980).
16. Y. Tsuji, Y. Morikawa, and O. Mizuno, J. Fluid Engineering, **4**, 484-488 (1985).
17. V. I. Klyatskin, Stochastic Equations and Waves in Randomly Inhomogeneous Media [in Russian], Moscow (1980).
18. I. V. Derevich and L. I. Zaichik, Prikl. Mat. Mekh., **54**, No. 5, 767-774 (1990).
19. I. N. Gusev and L. I. Zaichik, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 50-60 (1991).
20. K. Cercignani, Mathematical Methods in the Kinetic Theory of Gases [Russian translation], Moscow (1973).
21. I. V. Derevich and V. M. Eroshenko, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 69-78 (1990).